2.11 Momentum (1)

▲ Momentum = mass × velocity, and this truck has lots of it.

People say that a heavy vehicle travelling fast has lots of **momentum**. However, momentum has an exact scientific definition:

$$\text{momentum} = \text{mass} \times \text{velocity}$$

For example, if a model car has a mass of 2 kg and a velocity of 3 m/s, its momentum = mass × velocity = 2 kg × 3 m/s = 6 kg m/s

Like velocity, momentum is a vector, so a + or a − is often used to indicate its direction. For example:

momentum of car moving to the *right* = +6 kg m/s
momentum of car moving to the *left* = −6 kg m/s

Linking force and momentum: Newton's second law of motion

With a resultant force on it, an object will accelerate. Therefore, its velocity will change, and so will its momentum. The force and the momentum change are linked by this equation:

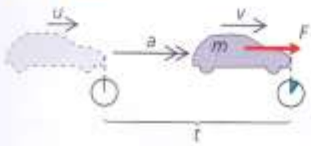
$$\text{resultant force} = \frac{\text{change in momentum}}{\text{time}}$$

or: resultant force = rate of change of momentum

The link between a resultant force and the rate of change of momentum it produces is known as **Newton's second law of motion**.

The above equation is really another way of saying that 'force = mass × acceleration'. The panel on the left explains why.

Two versions of the same law



A resultant force F acts on an object of mass m for a time t. As a result, its velocity increases from u to v, its acceleration over this time being a.

From Newton's second law of motion:

$$\text{resultant force} = \frac{\text{change in momentum}}{\text{time}}$$

So: $$F = \frac{mv - mu}{t} = m\left(\frac{v-u}{t}\right)$$

But: $$a = \left(\frac{v-u}{t}\right)$$

So: $F = ma$

In words:

resultant force = mass × acceleration

Impulse

From the previous equation, it follows that:

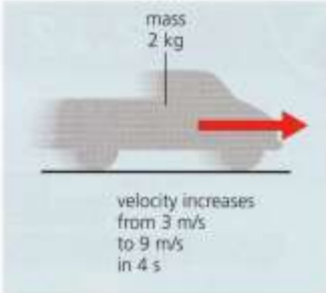
$$\text{resultant force} \times \text{time} = \text{change in momentum}$$

The quantity 'force × time' is called an **impulse**.

Newton noted that, when the same force acted for the same time on different masses, a large mass would gain less velocity than a smaller one, but the change in 'mass × velocity' was the same in every case. It was this observation that led to the concept of momentum and the second law.

Solving problems

Example 1 A model car of mass 2 kg is travelling in a straight line. If its velocity increases from 3 m/s to 9 m/s in 4 s, what is the resultant force on it?



To begin with:
momentum = mass × velocity = 2 kg × 3 m/s = 6 kg m/s

4 seconds later:
momentum = mass × velocity = 2 kg × 9 m/s = 18 kg m/s

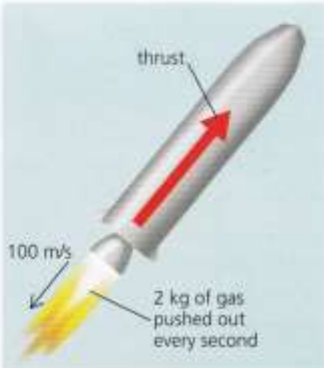
So: change in momentum = 12 kg m/s

But: $\text{resultant force} = \dfrac{\text{change in momentum}}{\text{time}} = \dfrac{12 \text{ kg m/s}}{4 \text{ s}}$

So: resultant force = 3N

The problem can also be solved by working out the car's acceleration and then using the equation: resultant force = mass × acceleration.

Example 2 A small rocket pushes out 2 kg of exhaust gas every second at a velocity of 100 m/s. What thrust (force) is produced by the engine?



By Newton's third law of motion, the forward force on the engine is equal to the backward force pushing out the exhaust gas. That force can be calculated by finding the rate of change of momentum of the gas:

In 1 second, 2 kg of gas increases its velocity from 0 to 100 m/s.

So: change in momentum = mass × velocity change
= 2 kg × 100 m/s = 200 kg m/s

$\text{force on gas} = \dfrac{\text{change in momentum}}{\text{time}} = \dfrac{200 \text{ kg m/s}}{1 \text{s}}$

So: thrust = 200 N

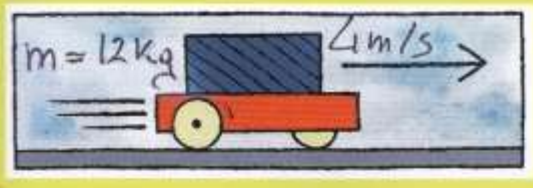
Q

1 What equation is used to calculate momentum?

2 What equation links the resultant force with the change in momentum it produces?

3 When a resultant force acts for 3 seconds on the trolley below, its velocity increases to 6 m/s.

- **a** What is the momentum of the trolley before the force acts?
- **b** What is the momentum after the force has acted?



- **c** What is the change in momentum?
- **d** What is the change in momentum every second?
- **e** What is the resultant force on the trolley?

Now you will calculate the resultant force on the trolley using different steps:

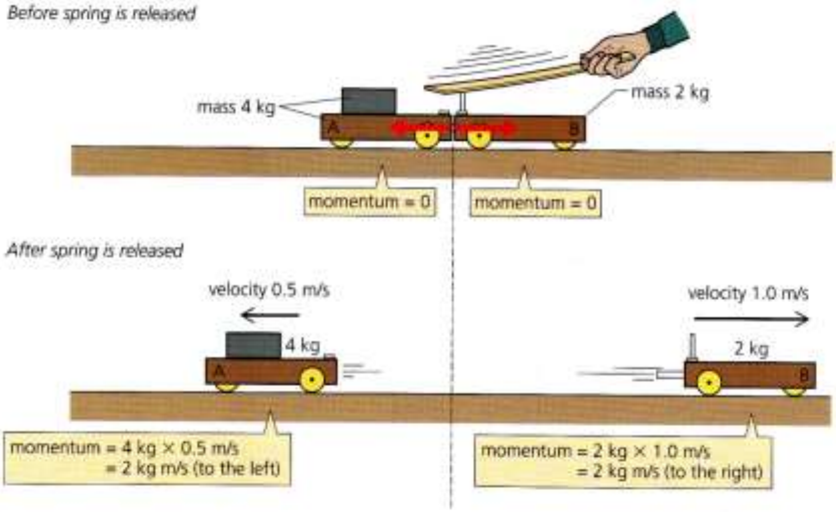
- **f** What is the trolley's change in velocity?
- **g** What is the trolley's acceleration?
- **h** What equation links force, mass, and acceleration?
- **i** What is the resultant force on the trolley?

4 A jet engine pushes out 50 kg of gas (mainly air) every second, at a velocity of 150 m/s.

- **a** What thrust (force) does the engine produce?
- **b** If the engine pushed out twice the mass of gas at half the velocity, what would the thrust be?

Related topics: velocity, acceleration as vectors **2.01**; force, mass, acceleration, Newton's 2nd law **2.08**; Newton's 3rd law **2.10**; momentum and molecules **5.05**

2.12 Momentum (2)



Velocity and momentum essentials

Velocity is speed in a particular direction.

momentum = mass × velocity
(kg m/s) (kg) (m/s)

Velocity and momentum are vectors. They have direction as well as magnitude (size). Their direction can be shown using an arrow, or a + or −.

To begin with, the trolleys above are stationary. But when a spring-loaded piston is released between them, they shoot off in opposite directions. Their velocities can be measured using ticker-tape timers.

When the trolleys shoot apart, the trolley with least mass has most velocity. The diagram shows typical mass and velocity values. These illustrate a rule which applies in all such experiments:

mass × velocity to the left = mass × velocity to the right
(trolley A) (trolley B)

This result is to be expected. From Newton's third law of motion, the forces on the two trolleys are equal but opposite. Also, the forces act for the same time. So they should cause equal but opposite changes in momentum (as force × time = change in momentum).

Conservation of momentum

With the mass and velocity values above, the total momentum of the trolleys before and after separation can be found. As momentum is a vector, its direction must be allowed for. In the following calculations, a momentum gain to the *right* is counted as *positive* (+):

Before the spring is released: total momentum of trolleys = 0

After the spring is released:

momentum of trolley A = mass × velocity = 4 kg × −0.5 m/s = −2 kg m/s

momentum of trolley B = mass × velocity = 2 kg × 1.0 m/s = +2 kg m/s

So: total momentum of trolleys = 0

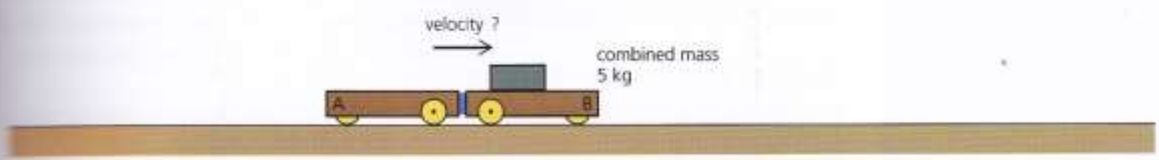
So the total momentum (zero) is unchanged by the release of the spring. This is an example of the **law of conservation of momentum**:

When two or more objects act on each other, their total momentum remains constant, provided no external forces are acting.

Collision problem

Before the collision

After the collision



Example When the two trolleys above collide, they stick together. What is their velocity after the collision?

According to the law of conservation of momentum, the total momentum of the trolleys is the same after the collision as before:

Before the collision:

momentum of trolley A = mass × velocity = 1 kg × 2 m/s = +2 kg m/s

momentum of trolley B = mass × velocity = 4 kg × −3 m/s = −12 kg m/s

So: total momentum of trolleys A and B = −10 kg m/s

After the collision:

total momentum of trolleys A and B = −10 kg m/s (as above)

So: combined mass × velocity = −10 kg m/s

So: 5 kg × velocity = −10 kg m/s

So: velocity of trolleys = −2 m/s

Therefore the trolleys have a velocity of 2 m/s to the left.

Momentum and energy

Moving objects have kinetic energy (see spread 4.01). In a collision, some of that energy may be changed into other forms.

If a collision is elastic, the total kinetic energy of the moving objects is the same after the collision as before. In other words, there is 'perfect bounce'. However, most collisions are not like this. The total kinetic energy is less after the collision than before. In such cases, the 'missing' energy is changed into heat (thermal energy).

Q

1 A trolley of mass 2 kg rests next to a trolley of mass 3 kg on a flat bench. When a spring is released between the trolleys, and they are pushed apart, the 2 kg trolley travels to the left at 6 m/s.

Before separation:

a What is the total momentum of the trolleys?

After separation:

b What is the total momentum of the trolleys?

c What is the momentum of the 2 kg trolley?

d What is the momentum of the 3 kg trolley?

e What is the velocity of the 3 kg trolley?

2 A 16 kg mass travelling to the right at 5 m/s collides with a 4 kg mass travelling to the left, also at 5 m/s. When the masses collide, they stick together and move along the same line as before.

Before the collision:

a What is the momentum of the 16 kg mass?

b What is the momentum of the 4 kg mass?

c What is the total momentum of the masses?

After the collision:

d What is the total momentum of the masses?

e What is the velocity of the masses?

Related topics: velocity and vectors **2.01**; using ticker-tape **2.04**; Newton's 3rd law **2.10**; kinetic energy **4.01–4.03**